Solver Coupling with Algebraic Constraints: An Index-2 Co-Simulation Approach

TECHNISCHE UNIVERSITÄT DARMSTADT

Daixing Lu, Pu Li & Bernhard Schweizer

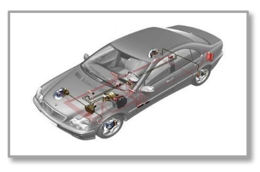
Institute of Applied Dynamics, Technical University of Darmstadt, Germany

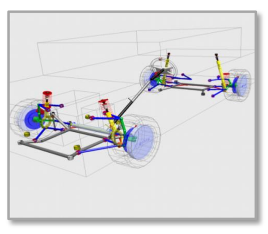
Introduction

- A **co-simulation method** on **index-2 level** is presented and analyzed with respect to numerical stability and convergence behavior.
- The coupling method is based on the **stabilized index-2 formulation** for multibody systems [1].
- The presented coupling technique is **semi-implicit** and based on a **predictor/corrector approach** [2, 3].

Application Field

- MBS/Matlab-coupling:
  ⇒ e.g. simulation of controlled automotive system
- MBS/Hydraulic-coupling:
  ⇒ e.g. simulation of servo-hydraulic steering system
- MBS/FEM-coupling:
  ⇒ e.g. simulation of pantograph-catenary system
- FEM/CFD-coupling:
  ⇒ e.g. simulation of fluid-structure interaction problem





General Idea of Co-Simulation:

Overall DAE-System

$C \cdot \dot{z} = f(z)$

Splitting

Subsystem 1

$C_1 \dot{z}_1 = f_1(z_1, \tilde{u}_2)$

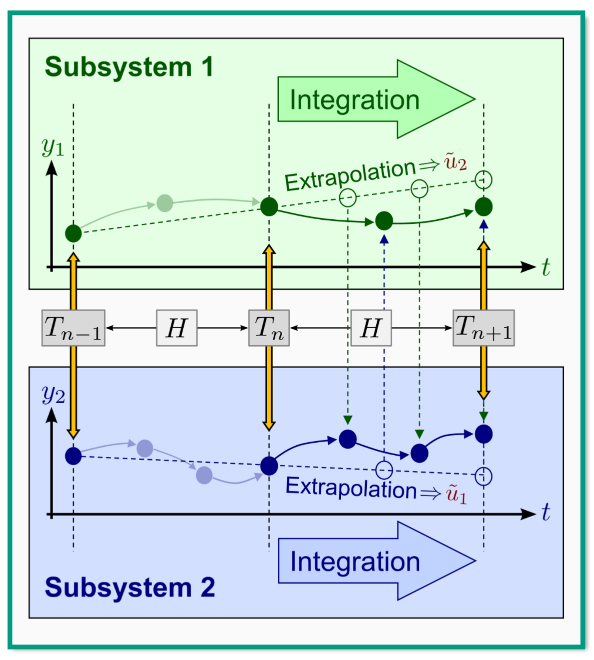
Coupling Conditions:

$g_c(\tilde{u}_1, \tilde{u}_2) = 0$

$C_2 \dot{z}_2 = f_2(z_2, \tilde{u}_1)$

Subsystem 2

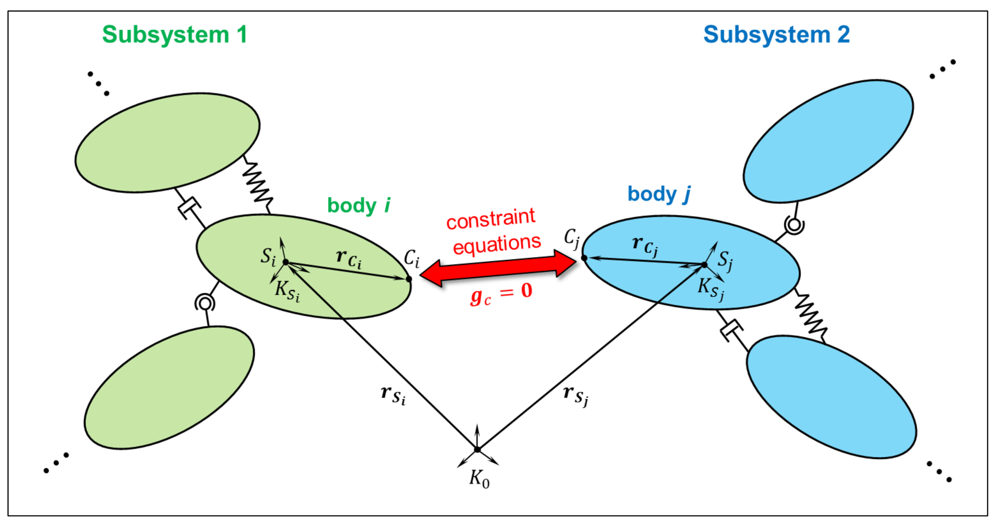
- Split overall system into 2 subsystems:
  => define coupling variables $\tilde{u}_1$ and $\tilde{u}_2$
  => couple subsystems by coupling conditions $g_c(\tilde{u}_1, \tilde{u}_2) = 0$



- Instead of a direct integration of the overall system:
  => separate integration of subsystems
  => subsystems are coupled by co-simulation approach

Co-Simulation

Here Considered: Two Multibody Systems Coupled by a Co-Simulation Approach



Subsystem 1

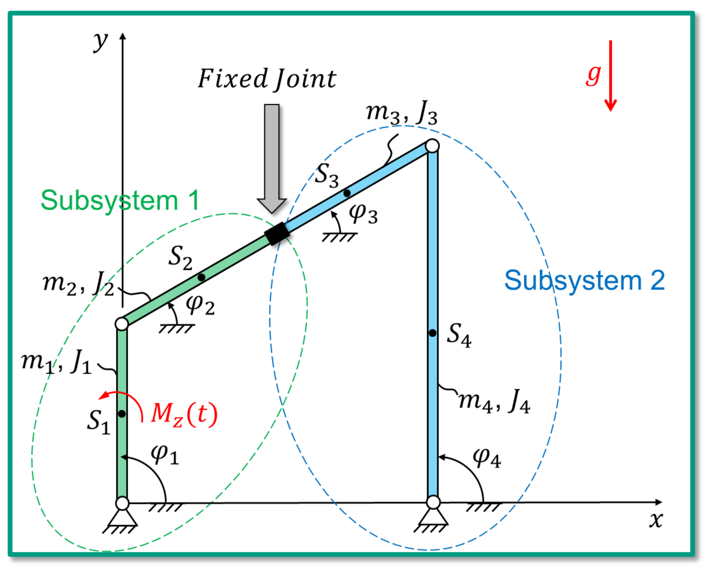
$$
\begin{aligned}
{}^0\dot{r}_{S_i} &= {}^0v_{S_i} + {}^0V_{c_i}\\
m_i\,{}^0\dot{v}_{S_i} &= {}^0F_{a_i}(\hat{z}_1, t) + {}^0F_{r_i} + {}^0F_{c_i}(z_c)\\
\dot{\gamma}_i &= B(\gamma_i)\,{}^i\omega_i + {}^i\Omega_{c_i}\\
({}^iJ_i\,{}^i\omega_i)^{\cdot} &= {}^iM_{a_i}(\hat{z}_1, t) + {}^iM_{r_i} + {}^iM_{c_i}(z_c)
\end{aligned}
$$

Subsystem 2

$$
\begin{aligned}
{}^0\dot{r}_{S_j} &= {}^0v_{S_j} + {}^0V_{c_j}\\
m_j\,{}^0\dot{v}_{S_j} &= {}^0F_{a_j}(\hat{z}_2, t) + {}^0F_{r_j} + {}^0F_{c_j}(z_c)\\
\dot{\gamma}_j &= B(\gamma_j)\,{}^j\omega_j + {}^j\Omega_{c_j}\\
({}^jJ_j\,{}^j\omega_j)^{\cdot} &= {}^jM_{a_j}(\hat{z}_2, t) + {}^jM_{r_j} + {}^jM_{c_j}(z_c)
\end{aligned}
$$

Numeric Experiment

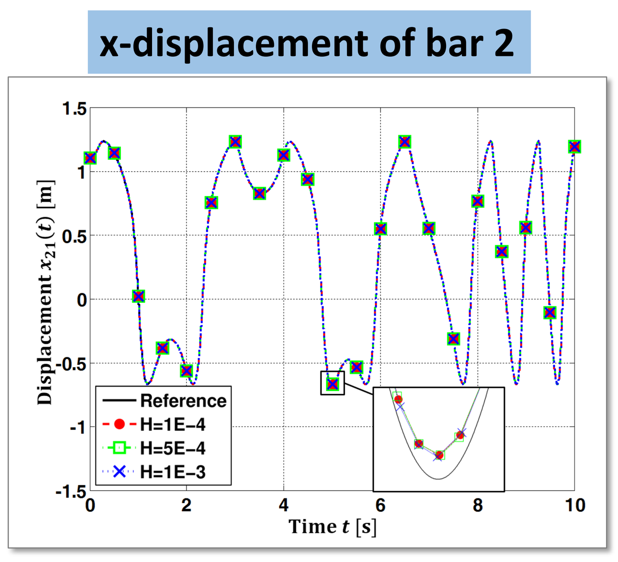
Planar 4-Bar Mechanism as Test Model:



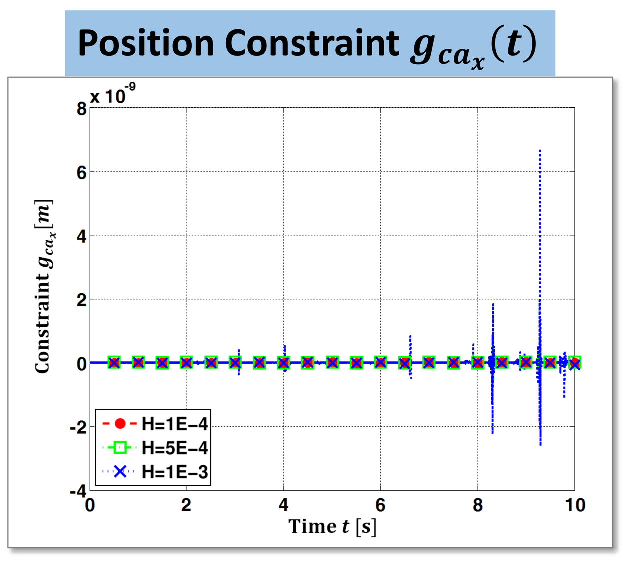
- **Decomposition** into 2 Subsystems
- **Coupling** of Subsystems by a fixed joint

Simulation Results with Stabilized Index-2 Co-Simulation Approach:

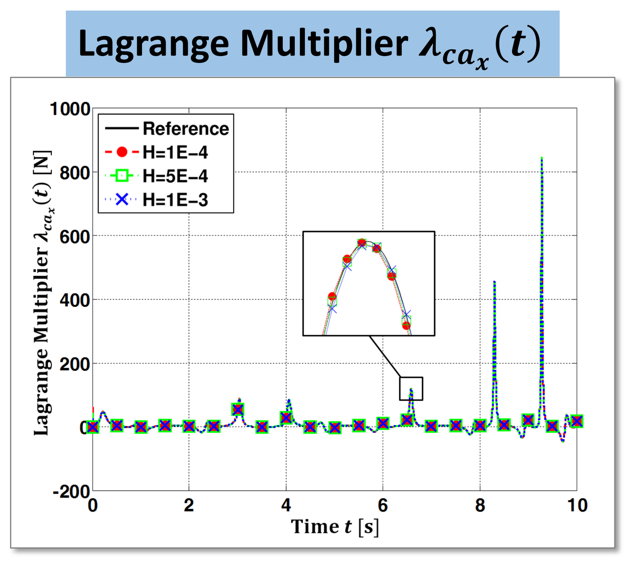
x-displacement of bar 2



Position Constraint $g_{ca_x}(t)$



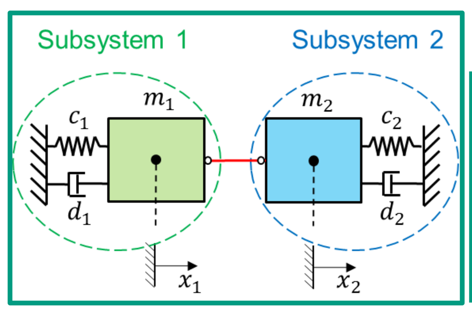
Lagrange Multiplier $\lambda_{ca_x}(t)$



- **Numerical error** in the position constraints is **smaller than 1E-8 m.**
- Coupling approach only introduces very **little numerical damping.**

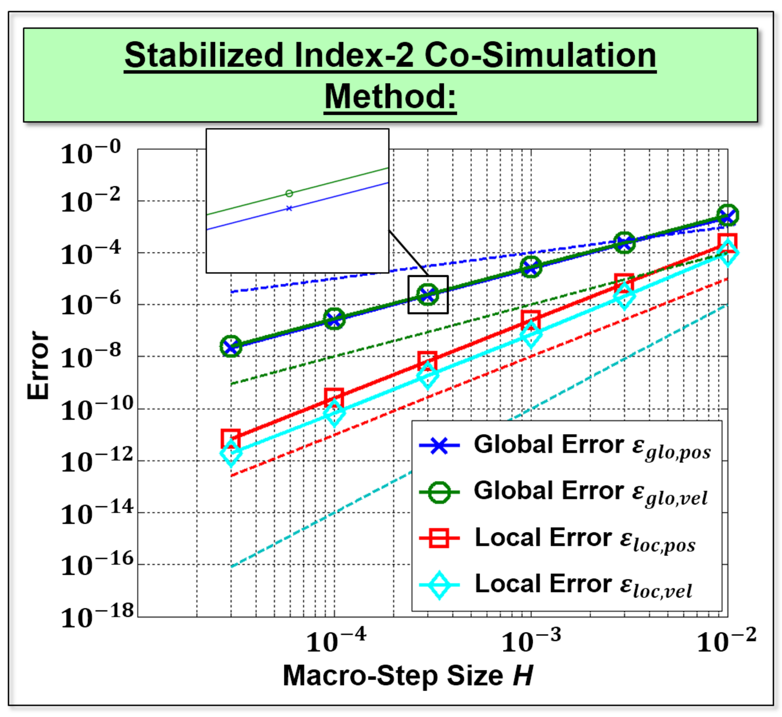
Stability and Convergence

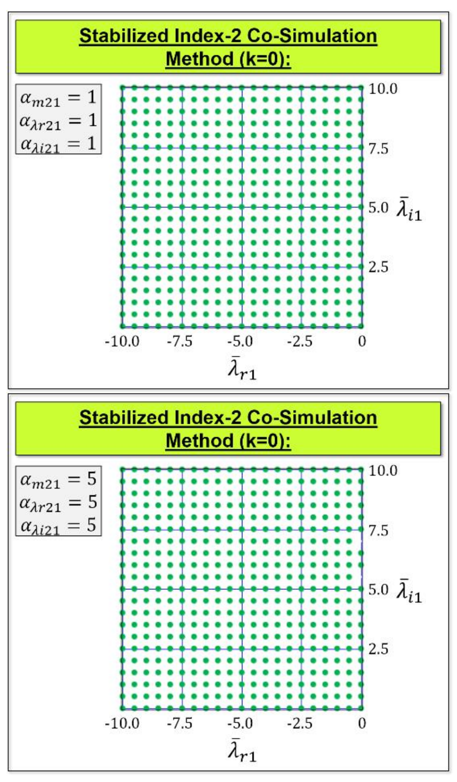
Test Model:



2-Mass-Oscillator
≙
2 algebraically coupled **Dahlquist´s** equations

- Global errors $\varepsilon_{glo,pos}$ and $\varepsilon_{glo,vel}$ of position and velocity variables:
  => convergence with $\mathcal{O}(H^2)$
- Local errors $\varepsilon_{loc,pos}$ and $\varepsilon_{loc,vel}$ of position and velocity variables:
  => convergence with $\mathcal{O}(H^3)$





Stability behavior:

- Fully stable for symmetric system
- Good stability for asymmetric systems

$\bar{\lambda}_{r1}$ and $\bar{\lambda}_{i1}$	… real and imaginary part of eigenvalue of subsystem 1
$\alpha_{m21} = \frac{m_2}{m_1}$	… mass ratio coefficient
$\alpha_{\lambda r21} = \frac{\bar{\lambda}_{r2}}{\bar{\lambda}_{r1}}$	… damping ratio coefficient
$\alpha_{\lambda i21} = \frac{\bar{\lambda}_{i2}}{\bar{\lambda}_{i1}}$	… frequency ratio coefficient

[1] Gear, C.W.; Gupta, G.K.; Leimkuhler, B.J.: “Automatic integration of the Euler-Lagrange equations”, J. Comp. Appl. Math., 12&13:77-90, 1985

[2] B. Schweizer, D. Lu. Stabilized index-2 co-simulation approach for solver coupling with algebraic constraints. Multibody System Dynamics, DOI: 10.1007/s11044-014-9422-y, 2014

[3] B. Schweizer, D. Lu, P. Li. Co-simulation method for solver coupling with algebraic constraints incorporating relaxation techniques, DOI: 10.1007/s11044-015-9464-9, 2015..